5.6 ENGINES

Combustion Gas Turbine (Brayton Cycle)

The typical approach for analysis of air standard cycles is illustrated by the Brayton Cycle in Fig. S-5.1. To understand the cycle, the basic idea is to write the balances for each step individually, then sum them up.

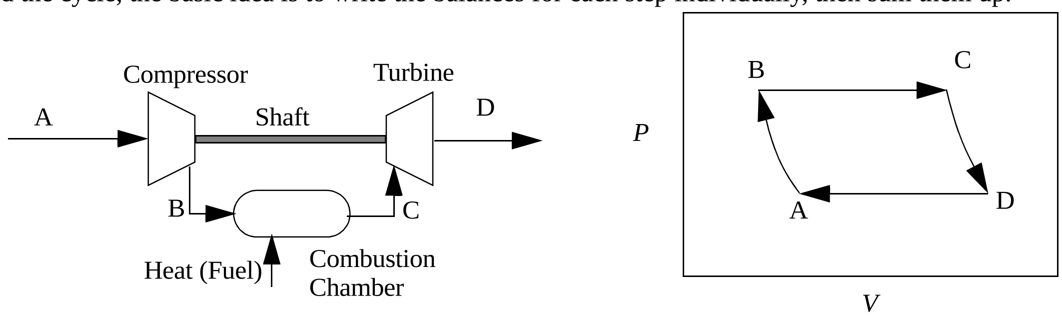

Figure S-5.1 *Gas turbine, and schematic of air-standard Brayton cycle.*

Example 5.6 Air-standard Brayton cycle thermal efficiency

Analyze the air-standard Brayton cycle to find the thermal efficiency and the dependence of temperature changes on pressure changes for an ideal gas across a reversible compressor and reversible turbine. Assume that the moles of gas produced by combustion are small relative to the total moles of gas throughput and can be ignored. Derive a relation for the thermal efficiency in terms of pressure ratio.

Solution: Refer to Fig. S-5.1 for stream labels,

System: closed packet of 1 mole of fluid passing through system.

$$A \rightarrow B \text{ compression:} \quad W_{S,AB} = \Delta H = C_P(T_B - T_A) \qquad (\text{*ig})$$

$$C \rightarrow D \text{ expansion:} \quad W_{S,CD} = \Delta H = C_P(T_D - T_C) \qquad (\text{*ig})$$

$$B \rightarrow C \text{ heat:} \quad Q = \Delta H = C_P(T_C - T_B) \qquad (\text{*ig})$$

$$\eta_\theta = \frac{-(W_{S,AB} + W_{S,CD})}{Q_{BC}} = \frac{-C_P\{(T_B - T_A) + (T_D - T_C)\}}{C_P(T_C - T_B)} + \left[1 - \frac{(T_C - T_B)}{(T_C - T_B)}\right]$$
$$= 1 - \left(\frac{T_D - T_A}{T_C - T_B}\right) \qquad (\text{*ig})$$

Where the $[1 - (T_C - T_B)/(T_C - T_B)] = 1 - 1$ has the net effect to write the efficiency relative to unity in a simplified form. The ratio of temperature differences in the result can be expressed in terms of pressure ratio. Noting that $P_D = P_A$ and $P_B = P_C$ permits us to express $P_D/P_C = P_A/P_B$. Substituting relations for adiabatic reversible ideal gases, $T'_D = T_C(P_A/P_B)^{R/C_P}$, $T_A = T'_B(P_A/P_B)^{R/C_P}$ and for a reversible process:

$$(T'_D - T_A) = \left(\frac{P_A}{P_B}\right)^{R/C_P}(T_C - T'_B) \quad \Rightarrow \quad \eta'_\theta = 1 - \left(\frac{P_A}{P_B}\right)^{R/C_P} \qquad (\text{*ig}) \text{ S-5.3}$$

Defining the specific heat ratio γ ($\equiv C_P/C_V$), the exponent is often replaced by $R/C_P = (\gamma - 1)/\gamma$. Therefore, efficiency is controlled by pressure ratio.

Turbofan Jet Engines

Many of the principles of the gas turbine are also present in turbofan jet aircraft engines. The engine also introduces fuel in a combustor. Like the gas turbine, the engine has a compressor system run by using some of the energy from the hot gases. The major difference is that, since the objective is to produce thrust, only a couple of turbine stages are used to drive the air compression system as shown in Fig. S-5.2. The gases exiting the turbine are at relatively high temperature and pressure, can be further heated in an afterburner, and create thrust by exiting the engine at high velocity through an exhaust nozzle. Afterburners consume fuel rapidly but are necessary for the highest thrust, and are turned on/off as needed. Afterburners are used almost exclusively on military aircraft and the Concorde.

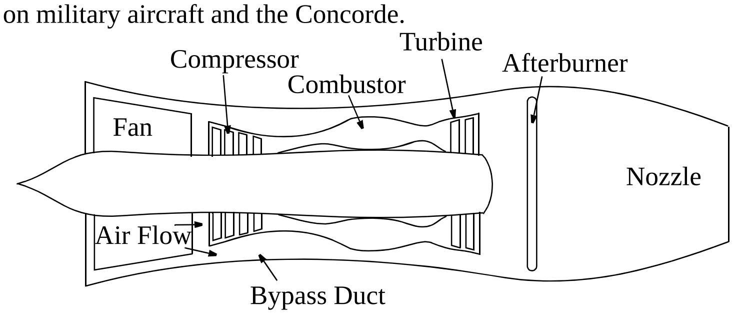

Figure S-5.2 *Schematic of a typical turbofan jet engine.*

There are two other features of a turbofan jet engine: 1) a large fan precedes the compressor on the same rotating shaft; 2) 40 to 97% of the low-pressure gas from the fan bypasses the compressor. Therefore, the overall zones in a turbofan jet engine are, from front to rear: the fan; the compressor with a surrounding bypass duct; the combustor that uses the high temperature/high pressure air from the compressor; a turbine, driven by the combustion gases to power the compressor; an afterburner to further heat the bypass gas and turbine outlet gases; an exhaust nozzle.

The Otto Cycle

Other kinds of air-based engines can be easily imagined. The Otto cycle describes the modern automobile engine. The actual *P-V* relations in an Otto engine follow a complex path of intake, compression, ignition/combustion (so rapid that $\Delta \underline{V} = 0$), expansion/work, and exhaust. Since the processes that occur in the Otto engine are extremely complex, a semiquantitative model has been developed using two adiabats and two isochoric steps shown in Fig. S-5.3. It is referred to as the air-standard Otto cycle.

Example 5.7 Thermal efficiency of the Otto engine

Determine the thermal efficiency of the air-standard Otto cycle as a function of the specific heat ratio γ (= C_P / C_V) and the compression ratio $\boldsymbol{r} \equiv V_1/V_2$. Assume an ideal gas, reversible cycle, and consider the moles generated by combustion to be negligible.

Solution: Basis: model as ideal gas,

$$Q_H = C_V(T_3 - T_2) \qquad \text{(*ig)}$$

$$Q_C = C_V(T_1 - T_4) \qquad \text{(*ig)}$$

$$-W_{S,net} = Q_H + Q_C = C_V(T_3 - T_2 + T_1 - T_4) \qquad \text{(*ig)}$$

$$\eta_\theta = C_V(T_3 - T_2 + T_1 - T_4)/\{C_V(T_3 - T_2)\} + [1 - (T_3 - T_2)/(T_3 - T_2)]$$

$$\eta_\theta = 1 - (T_4 - T_1)/(T_3 - T_2) \qquad \text{(*ig)}$$

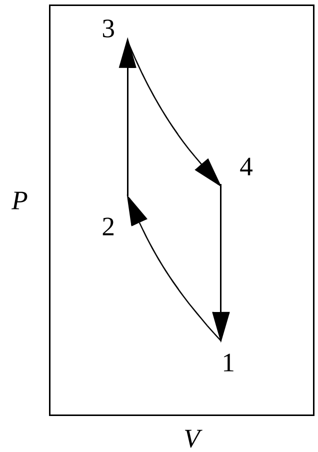

Figure S-5.3 *Schematic of the air-standard Otto cycle.*

Example 5.7 Thermal efficiency of the Otto engine (Continued)

Where the $[1-(T_3-T_2)/(T_3-T_2)] = 1-1$ has the net effect to write the efficiency relative to unity in a simplified form. Introducing the relations between temperature and volume for reversible adiabatic compression:

$$\frac{T'_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{R/C_V}; \quad \frac{T'_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{R/C_V} = \left(\frac{V_2}{V_1}\right)^{R/C_V} \qquad \text{(*ig)}$$

$$\Rightarrow T'_4 = T_3 r^{-R/C_V}; T_1 = T'_2 r^{-R/C_V} \Rightarrow (T'_4 - T_1) = r^{-R/C_V}(T_3 - T_2) \qquad \text{(*ig)}$$

$$\eta'_\theta = 1 - r^{-R/C_V} = 1 - r^{(1-\gamma)} \qquad \text{(*ig) S-5.4}$$

The Diesel Engine

The Diesel Engine is similar to the Otto engine except that the fuel is injected after the compression and the combustion occurs relatively slowly. This necessitates “fuel-injectors,” but has the advantage that higher compression ratios can be obtained without concern of pre-ignition (ignition at the wrong time in the cycle). Pre-ignition occurs when the temperature rise due to compression goes past the spontaneous ignition temperature of the fuel, creating an annoying pinging and knocking sound, and reducing the efficiency of the cycle. Pre-ignition does not occur in a diesel engine because there is no fuel during compression. Some spontaneous ignition temperatures are given below.

Spontaneous ignition temperatures (°C) of sample hydrocarbons

Isooctane	447
Benzene	592
Toluene	568
n-Octane	240
n-Decane	232
n-Hexadecane	230
Methanol	470
Ethanol	392

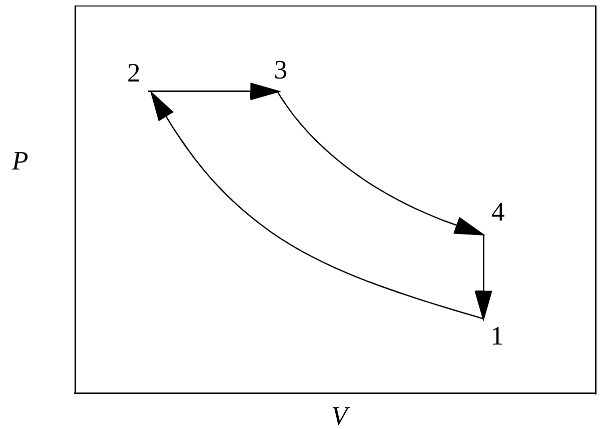

Figure S-5.4 *Schematic of air-standard Diesel cycle.*

Diesel fuel is much like decane and hexadecane and burns without a spark. Auto fuel is much like isooctane and benzene and, ideally, will not ignite until the spark goes off. But all fuels are mixtures, and if the gasoline contains enough *n*-octane instead of isooctane, then undesirable pre-ignition will occur.

The thermodynamics of the air-standard Diesel engine can be analyzed like the Otto Cycle. But, instead of rapid combustion at constant volume, the Diesel engine has relatively slow combustion at constant pressure. In the air-standard Diesel cycle shown in Figure S-5.4, step 1-2 is an adiabatic compression, step 2-3 represents the combustion where heat is added, step 3-4 is adiabatic, step 4-1 is an isochoric cooling.

Example 5.8 Thermal efficiency of a Diesel engine

Develop an expression for the thermal efficiency of the air-standard diesel cycle as a function of the compression ratio $r_c = V_1/V_2$ and the expansion ratio $r_e = V_4/V_3$. Assume the working fluid is an ideal gas, and the volume effect of moles of gas generated is small relative to the effect of heating from combustion.

Solution: This process is a little more complicated than the Otto cycle because heat addition and work occurs during constant pressure combustion. The energy balance for the combustion step is:

$$dU = Q_H + W_{S,combustion} \Rightarrow Q_H = \Delta U - W_{S,combustion} = \Delta U + P_2(V_3 - V_2) = \Delta(U + PV) = \Delta H$$

$$\Rightarrow Q_H = \Delta H = C_P(T_3 - T_2) \qquad \text{(*ig)}$$

For isochoric cooling

$$Q_C = C_V(T_1 - T_4) \qquad \text{(*ig)}$$

For the cycle, the energy balance is $0 = Q_H + Q_C + W_{S,net}$, giving the thermal efficiency

$$\eta_\theta = -W_{S,net}/Q_H = (Q_H + Q_C)/Q_H = 1 + \frac{C_V(T_1 - T_4)}{C_P(T_3 - T_2)} = 1 + \frac{1}{\gamma}\left[\frac{T_1}{T_3 - T_2} - \frac{T_4}{T_3 - T_2}\right] \qquad \text{(*ig) S-5.5}$$

The reciprocal of the first term in square brackets for a reversible device (ignoring the usual prime notation for all subsequent steps since all states are reversible),

$$\frac{T_3 - T_2}{T_1} = \frac{T_3}{T_1} - \frac{T_2}{T_1} = \frac{P_3V_3}{P_1V_1} - (r_c)^{\gamma-1} = (r_c)^{\gamma}/(r_e) - (r_c)^{\gamma-1} \qquad \text{(*ig)}$$

Example 5.8 Thermal efficiency of a Diesel engine (Continued)

where for T_2/T_1 we have used Eqn 2.64, and for T_3/T_1 we have used the ideal gas law, followed by $P_3 = P_2$ followed by Eqn. 2.65, and $V_1 = V_4$, as

$$\frac{T_3}{T_1} = \frac{P_3V_3}{P_1V_1} = \left(\frac{P_2}{P_1}\right)\left(\frac{V_3}{V_1}\right) = \left(\frac{V_1}{V_2}\right)^{\gamma}\left(\frac{V_3}{V_4}\right) = r_c^{\gamma}/r_e \quad \text{(*ig)}$$

For the reciprocal of last term in square brackets of S-5.5,

$$\frac{T_3 - T_2}{T_4} = \frac{T_3}{T_4} - \frac{T_2}{T_4} = (r_e)^{\gamma-1} - \frac{P_2V_2}{P_4V_4} = (r_e)^{\gamma-1} - \frac{P_3V_2}{P_4V_4} = (r_e)^{\gamma-1} - (r_e)^{\gamma}/(r_c) \quad \text{(*ig)}$$

where for T_3/T_4 we have used Eqn 2.63, and for T_2/T_4 we have used the ideal gas law, then $P_2 = P_3$, followed by Eqn. 2.65, and $V_4 = V_1$, as

$$\frac{T_2}{T_4} = \frac{P_2V_2}{P_4V_4} = \left(\frac{P_3}{P_4}\right)\left(\frac{V_2}{V_4}\right) = \left(\frac{V_4}{V_3}\right)^{\gamma}\left(\frac{V_2}{V_1}\right) = r_e^{\gamma}/r_c \quad \text{(*ig)}$$

Substuting the intermediate results and rearranging the formula for efficiency,

$$\begin{aligned} \eta'_{\theta} &= 1 + \frac{1}{\gamma}\left\{\frac{1}{(r_c)^{\gamma}/(r_e) - (r_c)^{\gamma-1}} - \frac{1}{(r_e)^{\gamma-1} - (r_e)^{\gamma}/(r_c)}\right\} \\ &= 1 + \frac{1}{\gamma}\left\{\frac{(r_e)}{(r_c)^{\gamma} - (r_e)(r_c)^{\gamma-1}} - \frac{(r_c)}{(r_c)(r_e)^{\gamma-1} - (r_e)^{\gamma}}\right\} \end{aligned} \quad \text{(*ig) S-5.6}$$