## S-A.1. ADVANCED FEATURES IN EXCEL

## Using Solver with Excel

Excel includes an "add-in" feature called Solver which can be used to solve single or multiple equations. The installation of the feature is optional. If it is installed, it will be listed under the Data ribbon in the Analysis group. A related feature is called Goal Seek..., but it is less powerful since it can only solve a single objective function. (Goal Seek is found under Data ribbon (Data Tools group)>What if analysis...). If Solver is not installed, use online help or find some directions at http://chethermo.net in the section for Software Tutorials.

To solve an equation for a single variable, Goal Seek or Solver can be used. We will use Solver since it is the more general tool, and use of Goal Seek is simple if desired, once Solver has been used. If you are using Solver with a spreadsheet that has been protected, it must first be unprotected using Review ribbon>Unprotect....

Suppose we wish to solve

$$
x^{2}+2 x=1
$$

Although the solution may be quickly found by the quadratic formula, a spreadsheet will be created to illustrate the technique that can be applied to more complex problems.

1. Create the following table, entering the labels in column $A$ and the initial guess of 0 for $x$ in cell $B 1$.

|  | A | B |
| :---: | :---: | :---: |
| 1 | x | 0 |
| 2 | $\mathrm{~F}(\mathrm{x})$ |  |

2. Enter the following formula in cell $\mathrm{B} 2:=\mathrm{B} 1 \wedge 2+2 * \mathrm{~B} 1-1$. (Note that you may click on cell B1 rather than typing the name as you enter the formula.)
3. Start Solver from the Data ribbon>Analysis Group. The Solver window will pop up as shown below in Figure S-A.1. The objective function (Target cell) is entered in the top entry box. For this example, enter B2 (or click in the entry box, then click in B 2 ). The radio buttons permit the objective cell to be maximized, minimized, or set to a specific value. In this example, select "Equal to: Value of:" and put the number 0 in the entry box. The Options... button controls the convergence criteria, but we won't use that now. (For more information on the options, search online help). The next entry box specifies the cells to adjust in the search for the objective function. For this example, enter B1. (For multiple cells you can drag the mouse over the cell ranges.)
4. Click on Solve. Look closely at the information box that pops up when Solver has finished. Since numerical techniques are used, Solver may have difficulty finding a solution for poor initial guesses or poorly defined objective functions, and the box notifies of problems in this event. However, we are solving a simple example here, and the answer is quickly found. The answer should be 0.414 .
5. If the other root is desired, a different initial guess should be entered in cell B1. Enter the value -2 and resolve the equation and be sure to uncheck the box to make the unconstrained variables non-negative. The answer -2.414 should result.

Excel may also be used to solve multiple equations. Consider the set of equations

$$
\begin{aligned}
& x^{2}+2 y=10 \\
& x+y=4
\end{aligned}
$$

Create the spreadsheet

|  | A | B | C | D |
| :---: | :---: | :---: | :---: | :---: |
| 1 | x | 1 | F1 |  |
| 2 | y | 3 | F2 |  |

The initial values in the second column satisfy the second equation. Let's call the first function F1. In cell D1 enter the formula $=\mathrm{B} 1 \wedge 2+2 * \mathrm{~B} 2-10$ and in cell D 2 enter $=\mathrm{B} 1+\mathrm{B} 2-4$. At this point some trial and error adjustment of the variable in column $B$ can help create a good initial guess, while judging the quality of the guess by watching column D. When you are ready, start Solver. Instruct Solver to Set D1 to a value of 0 by varying B1:B2 subject to the constraint D2 $=0$. (Click the Add... button to specify the constraint.) One solution is $\mathrm{x}=-0.732, \mathrm{y}=4.732$. Another solution is $\mathrm{x}=2.732$ and $\mathrm{y}=1.268$.

## Solving by Successive Substitution

Occasionally, solutions are needed to complex equations. For example, the equation $2.5 x-\exp (0.75 x)=0$ has two solutions. The solutions can be found by successive substitution. The technique works by rearranging the function in the form $x=f(x)$. For the example, there are two possibilities:

$$
\begin{equation*}
x=\frac{\exp (0.75 x)}{2.5} \tag{A}
\end{equation*}
$$

or

$$
\begin{equation*}
x=\frac{\ln (2.5 x)}{0.75} \tag{B}
\end{equation*}
$$



Figure S-A. 1 Dialog box for solver for one variable example.

Successive substitution works by using $f(x)$ to generate a new value of $x$ and then substituting back into $f(x)$. The technique will converge only if $\left|f^{\prime}(x)\right|<1$ at the value of $x$ guessed. For the present example, there are two solutions: $x=\{0.652536,2.37512\}$. To begin, enter the table into Excel.

|  | A | B |
| :---: | :---: | :---: |
| 1 | xnew $=\mathrm{f}(\mathrm{x})$ |  |
| 2 | xold | 1 |

Then for Eqn. A, enter into $B 1,=\exp \left(0.75^{*} B 2\right) / 2.5$. For the substitution, the following strategy is used: ${ }^{1}$

1. Copy cell B1.
2. Past the value into cell B2 by right-clicking on B2, selecting Paste Special... and then selecting the radio button for Paste Values. A macro can easily be created to repeat these steps as described below.

By repeating the substitution about 15 times, the iterations converge on $x=0.6525$. Try to converge on 2.37 by starting from 2.2 or 2.6 in cell B2-you will find it impossible because the value of $f^{\prime}(x)>1$ at $x=2.37$. Reprogram cell B1 to the right-hand side of Eqn. B. This time the successive substitution will converge to $x=2.375$, but convergence to $x=0.6524$ will be impossible.

## Macros for Successive Substitution

Successive substitution can be tedious, but a macro can be quickly written to perform the task. Suppose the following spreadsheet is available for the above example, solving for Eqn. A, where cell B1 is calculated from cell B2.

|  | A | B |
| :---: | :---: | :---: |
| 1 | xnew $=\mathrm{f}(\mathrm{x})$ | 0.8468 |
| 2 | xold | 1 |

Make sure the Developer tab is visible on the ribbon. By default, the Developer tab is not visible, so do the following:

1. Click the File tab, click Options and then click the Customize Ribbon category.
2. Under Customize the Ribbon, in the Main Tabs list, click Developer, and then click OK.

In the Code group on the Developer Ribbon, select Record new Macro... Give your macro a name and a shortcut keystroke. When you are ready to record, click OK. A Stop link will appear in the Ribbon in place of the Record link. Every key stroke or mouse click that is now entered will be recorded.

1. Copy cell B1.
2. Paste the value into cell B2 by right-clicking on B2, selecting Paste Special... and then selecting Paste Values.
3. Click the Stop Recording link from the Code group on the Developer Ribbon.

If all the steps went as planned, just press your shortcut key repeatedly. If you want to record over your macro, simply create a new macro with the same name.

## Naming Variables

Variables may be named for use in formulas. Select the cell range to be named (for an array, highlight the entire array range), then select the name box in the left-most edge of the formula bar. Type in the desired name. Default cell
1.Circular references can sometimes be created by entering $=\mathrm{B} 1$ into cell B 2 , after enabling iterations on the File $>$ Options $>$ Formulas $>$ Enable Iterative Calculation; however, the convergence or divergence can be unpredictable.
names cannot be used, e.g., X1 is invalid, but _X1 is acceptable. See "named ranges" in online help for more information.

## Array Operations

See Appendix B for an overview of matrices. To use a particular array element from an array, use the function INDEX(array,row_num,col_num). If the following named matrices exist in Excel:
when $B=\left[\begin{array}{lll}1 & 2 & 3 \\ 4 & 5 & 6\end{array}\right]$, then $\operatorname{INDEX}(B, 1,3)=3, \operatorname{INDEX}(B, 2,3)=6$
when $X=\left[\begin{array}{ll}0.4 & 0.6\end{array}\right]$, then $\operatorname{INDEX}(X, 1)=0.4, \operatorname{INDEX}(X, 2)=0.6$

When entering a formula for a matrix operation that will involve matrix multiplication or results in an array, select the entire array output range (with the correct number of rows and columns) before entering the formula, then press CNTL+SHIFT+ENTER (or COMMAND+RETURN on a Macintosh) to tell Excel that this is a matrix formula. Excel inserts braces around the formula after it is entered. Failure to select the correct number of output rows or columns, or failure to press CNTL+SHIFT+ENTER, or failure to observe the requirements of matching the number of rows and columns in multiplicands can result in unexpected results or errors. Some common operations are shown below. See SUM and MMULT in online help for more details.

One-dimensional arrays:
(a) If both arrays appear as columns or both arrays appear as rows, use SUM(array1*array2);
(b) If array1 is a row and array2 is a column, use MMULT(array1,array2);
(c) If array1 is a column and array2 is a row, use MMULT(array2,array1).

Multidimensional arrays (select the entire output range before entering the formula):
(a) For $c_{i j}=\sum_{k} a_{i k} b_{k j}$, use MMULT(A,B);
(b) For $c_{i j}=\sum_{k} a_{i k} b_{j k}$, use MMULT(A,TRANSPOSE(B)).

