5.7 FLUID FLOW

Consider the general flow system of Fig. S-5.5*a* in which work and heat are transferred and the fluid undergoes changes in kinetic and potential energy. Recognize that the compressor or pump in the schematic could be replaced with an expander or turbine. Rather than deriving an integral equation between points 1 and 4 in the schematic, let us consider a balance over a differential element at steady-state as shown in Fig. S-5.5*b* where the possibility of heat and work transfer are permitted. The steady-state balance for a single stream becomes:[1]

$$0 = \lim_{dL \to 0}\left\{\left[H + \frac{u^2}{2g_c} + \frac{gz}{g_c}\right]^{in} - \left[H + \frac{u^2}{2g_c} + \frac{gz}{g_c}\right]^{out}\right\} + dQ + dW_S$$

For a differential element, $H^{out} = H^{in} + dH$, and

$$\left(\frac{u^2}{2g_c}\right)^{out} = \left(\frac{u^2}{2g_c}\right)^{in} + d\left(\frac{u^2}{2g_c}\right), \quad \left(\frac{gz}{g_c}\right)^{out} = \left(\frac{gz}{g_c}\right)^{in} + d\left(\frac{gz}{g_c}\right)$$

The differential balance becomes

$$0 = -dH - d\left(\frac{u^2}{2g_c}\right) - d\left(\frac{gz}{g_c}\right) + dQ + dW_S$$

Recognizing $d\left(\frac{u^2}{2g_c}\right) = \frac{udu}{g_c}$, and over practical distances g is constant, resulting in $d\left(\frac{gz}{g_c}\right) = \frac{g}{g_c}dz$.

$$(dH - dQ) + \frac{udu}{g_c} + \frac{g}{g_c}dz = dW_S$$

The entropy balance for the differential system of Fig. S-5.5*b*,

$$0 = \lim_{dL \to 0}\{S^{in} - S^{out}\} + \frac{dQ}{T} + dS_{gen}$$

and $S^{out} = S^{in} + dS$,

$$TdS - TdS_{gen} = dQ$$

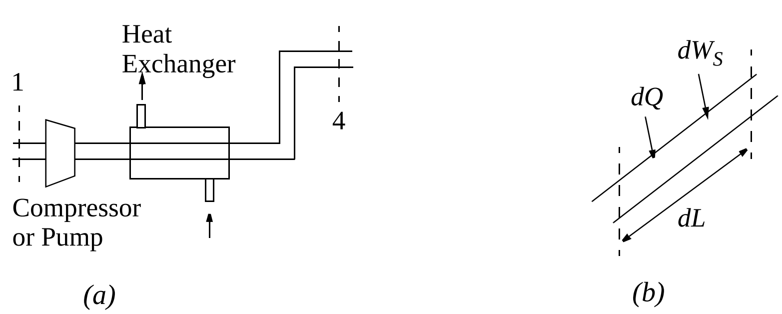

Figure S-5.5 *(a) Schematic of a general overall system, and (b) differential balance*

1. Note that the velocity in this equation is a mean velocity.

Combining with the energy balance,

$$(dH - TdS) + TdS_{gen} + \frac{udu}{g_c} + \frac{g}{g_c}dz = dW_S$$

Noting the fundamental relation for enthalpy, $dH = TdS + VdP$,

$$VdP + TdS_{gen} + \frac{udu}{g_c} + \frac{g}{g_c}dz = dW_S$$

Although *H* and *S* are state properties, *TdS* is path-dependent, therefore *VdP* is path-dependent and must be evaluated along the actual path. The term TdS_{gen} represents the losses due to viscosity, and is often called the lost work, *lw*.

Lost work is due to entropy generation.

$$\boxed{TdS_{gen} \equiv d(lw)} \tag{S-5.7}$$

The energy balance becomes

The differential energy balance.

$$\boxed{VdP + d(lw) + \frac{udu}{g_c} + \frac{g}{g_c}dz = dW_S} \tag{S-5.8}$$

The treatment of *d*(*lw*) depends on the device. Typically, the differential balance is integrated over the individual pieces of equipment in Fig. S-5.5*a* in the three categories: 1) pipes or fittings, $(lw)_f$; 2) pumps or compressors, $(lw)_p$; 3) expanders or turbines, $(lw)_t$.

Pipes

The frictional losses in pipes can be predicted by an empirical variable *f* known as the Fanning friction factor, and the losses due to fittings or sudden cross-section changes can be handled separately

$$d(lw)_f = \frac{2fu^2}{g_c D}dL + d(lw)_{fittings} \tag{S-5.9}$$

where *D* is the diameter of the pipe and $d(lw)_{fittings}$ is the lost work due to fittings and sudden cross-section changes. The friction factor is relatively insensitive to considerable temperature changes. Work producing/generating devices are not present in flow through pipes, $dW_S = 0$,

General differential flow in a pipe.

$$VdP + \frac{2fu^2}{g_c D}dL + d(lw)_{fittings} + \frac{udu}{g_c} + \frac{g}{g_c}dz = 0 \tag{S-5.10}$$

For an incompressible fluid in a pipe, the density changes will be negligible. Integrating term by term, where ρ is the mass density,

Flow of an incompressible fluid.

$$\boxed{\frac{\Delta P}{\rho} + \frac{2fu^2L}{g_c D} + (lw)_{fittings} + \frac{\Delta u^2}{2g_c} + \frac{g}{g_c}\Delta z = 0} \tag{S-5.11}$$

For a compressible fluid such as a gas, velocity and density can change appreciably with pressure and temperature. The mass flowrate is constant at steady state,

$$\dot{m} = \frac{Au}{V} = AG \tag{S-5.12}$$

where A is the cross-sectional area of the pipe, $G \equiv u/V = u\rho$, and G is called the mass velocity and is constant when A is constant. Using $u = G/\rho$, $du = (-G/\rho^2)d\rho$. Substituting into Eqn. S-5.10 for a horizontal pipe without fittings or sudden cross-section changes,

$$\frac{dP}{\rho} + \frac{2fG^2}{g_c\rho^2 D}dL - \frac{G^2}{g_c\rho^3}d\rho = 0\,, \quad \rho dP + \frac{2fG^2}{g_c D}dL - \frac{G^2}{g_c}\frac{d\rho}{\rho} = 0$$

For an ideal gas,

$$(MW)\int \frac{P}{RT}dP + \frac{2fG^2L}{g_cD} - \frac{G^2}{g_c}\ln\frac{\rho_2}{\rho_1} = 0 \qquad \text{(ig)}$$

where (MW) is the molecular weight. If the flow is isothermal,

$$\boxed{\frac{(MW)(P_2^2 - P_1^2)}{2RT} + \frac{2fG^2L}{g_cD} - \frac{G^2}{g_c}\ln\frac{P_2}{P_1} = 0} \qquad \text{(ig) S-5.13}$$

Isothermal flow of an ideal gas in a horizontal pipe of constant size and no fittings.

The Fanning friction factor, f, depends on the properties of the fluid and the size and roughness of the pipe. The friction factor is most easily characterized in terms of the dimensionless Reynolds number, $Re = Du\rho/\mu$, where D is the diameter of the pipe, ρ is the fluid mass density, and μ is the fluid viscosity. The relationships for smooth pipes are:

$$f = \frac{16}{Re} \text{ for } Re < 2100 \qquad \text{S-5.14}$$

$$f = 0.0014 + \frac{0.125}{Re^{0.32}} \text{ for } 3000 < Re < 3.\text{ E } 6 \qquad \text{S-5.15}$$

Below Re = 2100 the flow is laminar; above 4000 the flow is turbulent. The range of Reynolds number between 2100 and 4000 is known as the transition region where the friction factor correlations are somewhat uncertain because the flow is in between the laminar and turbulent flow regimes. Fluid mechanics textbooks can be consulted for calculating flows in rough pipes, fittings, sudden contractions or expansions, non-circular pipes, or non-isothermal conditions.

Pumps or Compressors

The lost work due to pump/compressor irreversibility, $(lw)_p$, is incorporated into the energy balance by the pump/compressor efficiency

$$\eta = \frac{W_S - (lw)_p}{W_S} \text{ where } W_S > 0,\ (lw)_p > 0 \qquad \text{S-5.16}$$

W_S is the actual work transferred by the pump/compressor. Therefore, the terms $W_S - (lw)_p$ in the energy balance can be replaced by ηW_S. For a flow system including piping for an *incompressible* fluid, Eqn. S-5.11 becomes

$$\boxed{\frac{\Delta P}{\rho} + \frac{2fu^2L}{g_cD} + (lw)_{fittings} + \frac{\Delta u^2}{2g_c} + \frac{g}{g_c}\Delta z = \eta W_S} \qquad \text{S-5.17}$$

Pumping of an incompressible fluid.

By analogy, an *isothermal* pump or compressor for a *compressible* fluid will result in ηW_S on the right-hand side of Eqn. S-5.13.

Turbines or Expanders

The lost work due to turbine/expander irreversibilities $(lw)_t$, is incorporated by the turbine/expander efficiency

$$\eta = \frac{W_S}{W_S - (lw)_t} \text{ where } W_S < 0, (lw)_t > 0. \qquad \text{S-5.18}$$

W_S is the actual work interaction of the fluid with the turbine/expander. Therefore the terms $W_S - (lw)_t$ in the energy balance may be replaced with W_S/η. By analogy with Eqn. S-5.17 for an *incompressible* fluid

Turbine for an incompressible fluid.

$$\boxed{\frac{\Delta P}{\rho} + \frac{2fu^2L}{g_cD} + (lw)_{fittings} + \frac{\Delta u^2}{2g_c} + \frac{g}{g_c}\Delta z = \frac{W_S}{\eta}} \qquad \text{S-5.19}$$

A similar modification may be made to Eqn. S-5.13 for an *isothermal* turbine/expander for a compressible fluid.