(P2.1) Energy Balance: Eqn. 2.33
$\frac{d}{d t}\left[m\left(U+\frac{v^{2}}{2 g_{C}}+\frac{g Z}{g_{C}}\right)\right]=\sum_{\text {inlets }}\left[H+\frac{v^{2}}{2 g_{C}}+\frac{g Z}{g_{C}}\right]^{\text {in }} \dot{m}^{\text {in }}-\sum_{\text {outlest }}\left[H+\frac{v^{2}}{2 g_{C}}+\frac{g z}{g_{C}}\right]^{\text {out }} \dot{m}^{\text {out }}+\underline{\dot{Q}}+\underline{\dot{W}}_{E C}+\underline{\dot{W}}_{S}$

closed system, no mass flow $\Rightarrow \dot{m}^{\text {in }}=\dot{m}^{\text {out }}=0$, valve stuck preventing any steam from going out $\Rightarrow \dot{m}^{\text {out }}=0$, no change in volume
occurs (no expansion or contraction) $\Rightarrow \underline{W}_{E C}=0$, no work has been added
or removed, and there is no pump nor turbine $\Rightarrow \underline{W}_{S}=0$.
Ignore K.E. and P.E.
$\Rightarrow$ the final energy balance becomes. $\frac{d}{d t}(m U)=\underline{\dot{Q}}$

(P2.3) (Solution for gas furnace). Assume the house is air-tight. Furnace has been heating the home steadily, $\Rightarrow$ the system (furnace) is a steady-state open system, $\Rightarrow$
$\frac{d}{d t}\left[m\left(U+\frac{v^{2}}{2 g_{C}}+\frac{g Z}{g_{C}}\right)\right]=0 \quad \therefore \dot{m}^{\text {in }}=\dot{m}^{\text {out }}$,
Furnace is fixed size, no expansion/contraction work occurs. Consider just the hot side of the heat exchanger.

$$
\therefore \dot{m}^{\text {out }} H^{\text {out }}-\dot{m}^{\text {in }} H^{\text {in }}=\underline{Q}
$$

(Chap 3 will consider reactions such as

combustion, but the E-balance doesn't need to be written differently!)
Note: if the small shaft work from blower is included, then the boundary includes the house air in and out of the furnace, and the boundary includes both sides of the heat

## Chapter 2 Practice Problems

exchanger. For this boundary, all heat transfer is within the furnace, and there is no Q term, but there are four mass flow terms (fuel/air in, combustion gas out, house air in, house air return) and a small (negligible compared to other terms) Ws term.
(P2.4) (Solution for gas furnace.) Assume house is air-tight, $m^{\text {in }}=m^{\text {out }}=0$.
System: the house and all contents, consider just the house side of heat exchanger $\Rightarrow$ closed, unsteady-state system.
$\Rightarrow$ there is heat gain and heat lost in the house.
No expansion / contraction work nor kinetic, potential energy change.
The internal energy of the system could change with respect to time depending on heat gain and heat loss, or could be zero if they are balanced. If the furnace is properly sized, there will be a net gain in temperature of the house!


$$
\frac{d}{d t}(m U)=\dot{Q}_{\text {gain }}+\dot{\underline{Q}}_{\text {loss }}, \text { where } \underline{\dot{Q}}_{\text {loss }} \text { is negative. }
$$

Note: if blower is included within system, then you also should be clear whether the boundary includes the entire heat exchanger (in which case the $\mathrm{Q}_{\text {gain }}$ term is replaced with enthalpy flow terms), or just the house side of the heat exchanger. In either event, the Ws term will be small relative to other terms since there is so much heat coming from furnace.
(P2.5) System is child:
no mass flow across boundary of child. ignore heat transfer from cold snow. impact of snow is a surface work that is interpreted as shaft work.

we might now think of the snow as a system. Again ignoring heat transfer from melting, and ignoring potential energy change of snow and any melting or T change of snow,

$$
\Delta\left[\not p U+\frac{m v^{2}}{2 g_{C}}+\frac{m g z}{g_{C}}\right]_{\text {snow }}=\not \dot{\phi}^{\hat{W}}+\mathscr{W}_{E C}+\underline{W}_{S, \text { snow }},
$$

The work terms are equal and opposite

$$
\Rightarrow \Delta\left[m U+\frac{m v^{2}}{2 g_{C}}\right]_{\text {child }}=-\left[\frac{m \Delta v^{2}}{2 g_{C}}\right]_{\text {snow }}
$$

(P2.6) System: the bulb and its contents.
System is closed steady state system, $\Rightarrow \frac{d}{d t}(\forall)=0 \ldots . \forall$ is left side of equation.
There is no work acting on the bulb and no mass flow in or out.
$\Rightarrow \Delta \underline{U}=0$


Consider photons as heat transfer by radiation $\dot{\underline{Q}}_{\text {photons }}$ and heat loss from bulb, $\dot{\underline{Q}}_{\text {bulb }}$,

$$
\underline{\dot{Q}}_{\text {net }}=0
$$

(P2.7) System: the sunbather at 12:00 noon (open system).
No work has been done, no kinetic or potential energies.
No mass flow in, but perspiration loss, $\dot{m}_{b o d y}=-\dot{m}^{\text {out }}$


Unsteady state, still getting hot.
Consider photons to be heat transfer by radiation.
$\Rightarrow$ by using the equation 2.64 (text book) and drop out all the zero values,
$\Rightarrow$ energy balance will be: $\frac{d}{d t}\left(m_{b o d y} U_{b o d y}\right)=\dot{m}_{b o d y} H^{V}+\underline{\dot{Q}}$
(steam tables could be used to estimate the enthalpy of the evaporating perspiration if the T is known or estimated).
(P2.8) System: the balloon and its contents.
Unsteady state system, open system.
No heat gain or loss.
No shaft work.


Mass changing with respect to time, as it gets flat, $\dot{m}=-\dot{m}^{\text {out }}$
Volume of the balloon is changing the work of expansion / contraction occurs.
Kinetic energy effected by changing the velocity of the balloon.
No potential energy change.
$\Rightarrow$ energy balance is: $\frac{d}{d t}\left[m U+\frac{m v_{\text {balloon }}^{2}}{2 g_{C}}\right]=+\left[H+\frac{v^{2}}{2 g_{C}}\right]^{\text {out }} \frac{d m}{d t}+\underline{\underline{W}}_{E C}$
(P2.9) (a) Potential energy (P.E), $\Rightarrow \Delta \mathrm{PE}=\mathrm{m}^{*} \mathrm{~g}^{*} \Delta \mathrm{~h},(1 \mathrm{~J}=1 \mathrm{Nm})$
where $m=$ mass,
$\mathrm{g}=$ gravity,
$\mathrm{h}=$ height or distance that the body moved by.
$\Rightarrow \Delta P \cdot E=m g \Delta h / g_{c}, \Rightarrow \Delta h=\frac{\Delta P \cdot E}{m g / g_{c}}=\frac{1000 \mathrm{Nm}}{1 \mathrm{~kg} * 9.8066 \mathrm{~N} / \mathrm{kg}}$
$\Rightarrow h=101.97 m$

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This is why potential energy is often ignored when thermal changes are present- the height change must be very large to be significant.
(b) Kinetic energy = potential energy
$\Rightarrow K . E=\frac{1}{2 g_{C}} m v^{2}, \Rightarrow u=\left(\frac{2 g_{c} K . E}{m}\right)^{\left(\frac{1}{2}\right)}=\left(\frac{2 * 1\left(\mathrm{kgm} / \mathrm{Ns}^{2}\right) * 1000 \mathrm{Nm}}{1 \mathrm{~kg}}\right)^{1 / 2}$
$\therefore v=44.72 \mathrm{~m} / \mathrm{s}$
This is why kinetic energy is often ignored when thermal changes are present- the velocity change must be very large to be significant.
(P2.10)Closed system, unsteady-state, 1 block, 2 water, 3 tank.
Energy balance $\Delta \underline{U}=0$
$\mathrm{m} 1^{*} \Delta \mathrm{U} 1+\mathrm{m} 2 * \Delta \mathrm{U} 2+\mathrm{m} 3 * \Delta \mathrm{U} 2=0$
$\left(\Delta \mathrm{U}=\int_{T 1}^{T 2} C v d T\right), \Delta U=C v \Delta T$

$(\mathrm{T}-400) * 200 * 0.380+4.184 * 4000 *(\mathrm{~T}-300)+500 * 0.380 *(\mathrm{~T}-300)=0$
Solve for $\mathrm{T}, \Rightarrow \mathrm{T}=300.447 \mathrm{~K}$
$\Delta \mathrm{U}_{\text {BLOCK }}=200 * 0.38 *(400-300.447)=-7566 \mathrm{~J}$
$\Delta \mathrm{U}_{\text {WATER }}=4000 * 4.184 *(300.447-300)=7480.9 \mathrm{~J}$
(P2.11) Unsteady-state, closed system, Energy balance with block at 50 m deep in the water.

$$
\begin{aligned}
& \mathrm{m} 1^{*} \Delta \mathrm{U} 1+\mathrm{m}^{*} \Delta \mathrm{U} 2+\mathrm{m}^{*} \Delta \mathrm{U} 2+\mathrm{m} 4(\mathrm{~kg})^{*} \Delta \mathrm{z}(\mathrm{~m})^{* g} / \mathrm{gc}(\mathrm{~N} / \mathrm{kg})=0 \quad\left(1 \mathrm{~J}=1 \mathrm{kgm}^{2} / \mathrm{s}^{2}\right) \\
& 200^{*} .38^{*}(\mathrm{~T}-400)-0.2^{*} 50 * 9.81+4000^{*} 4.184^{*}(\mathrm{~T}-300)+500^{*} .38^{*}(\mathrm{~T}-300)=0 \Rightarrow \\
& \mathrm{~T}=300.453 \quad \Rightarrow \Delta \mathrm{U}_{\text {BLOCK }}=-7566
\end{aligned}
$$

(P2.12) a. Unsteady-state closed system

$$
W_{E C}=0(\text { rigid }) \quad \Delta U=Q
$$

$\Delta \underline{U}_{\text {gas }}=\mathrm{nCv} \Delta \mathrm{T}=5 \mathrm{~mole} * 200 * 5 \mathrm{cal} / \mathrm{mol}-\mathrm{K}=5000 \mathrm{cal} . ;$ with gas AND vessel
$\Delta \underline{U}=n_{\text {gas }} C v_{\text {gas }} \Delta T+m_{\text {vessel }} C v_{\text {vessel }} \Delta T=$
$5000+m_{\text {vessel }} C v \Delta T=5000+80(0.125) 200=7000 \mathrm{cal}$
b. $\Delta \underline{U}=\underline{Q}+\underline{W}_{E C} \Rightarrow \Delta \underline{U}-\underline{W}_{E C}=\underline{Q} \Rightarrow \Delta \underline{H}=\underline{Q}=m C p \Delta T$

5mole*200*7cal/mol-K = 7000cal

(P2.13) Unsteady state, closed system.

Assume Ideal Gas. $\Rightarrow \mathrm{Cp}=\mathrm{Cv}+\mathrm{R}$, and $\mathrm{Cp}=7 \mathrm{R} / 2$ (diatomic)
$\Rightarrow 7 \mathrm{R} / 2-\mathrm{R}=\mathrm{Cv} ; \quad \Rightarrow \mathrm{Cv}=5 \mathrm{R} / 2=5 * 8.314 / 2=20.785 \mathrm{~J} / \mathrm{g}-\mathrm{k}$
$\mathrm{Q}=\Delta \mathrm{U}+\mathrm{W} \quad$ ( no work )
$\Rightarrow \mathrm{Q}=\Delta \mathrm{U}=20.785^{*}(400-300)$
$\Rightarrow \mathbf{Q}=2079 \mathrm{~J} / \mathrm{mol}$
(P2.14) Steady state, open system.
$660^{\circ} \mathrm{F}=348.88^{\circ} \mathrm{C}$
$\mathrm{T}\left({ }^{\circ} \mathrm{C}\right)=\left(\mathrm{T}\left({ }^{\circ} \mathrm{F}\right)-32\right) * 5 / 9$
Through the valve $\Rightarrow 0=\dot{m}^{\text {in }} H^{\text {in }}-\dot{m}^{\text {out }} H^{\text {out }} \Rightarrow H^{\text {in }}=H^{\text {out }}$
From steam tables (back of the book).
Interpolate for $348.88^{\circ} \mathrm{C}$ to find $\mathrm{H}^{\text {in }}$
$\Rightarrow \frac{350-348.88}{350-345}=\frac{2563.64-H}{2563.64-2594.9} \quad \Rightarrow \mathbf{H}^{\text {in }}=\mathbf{2 5 7 0 . 6 4} \mathbf{~ J} / \mathbf{g}$
Outlet at 1 atm. Use sat T table to find $\mathrm{P}=1.014 \mathrm{MPa}$ (close enough to 1 atm ). $\mathrm{H}^{\text {out }}$ is less than $\mathrm{H}^{\text {satV }}=2676 \mathrm{~J} / \mathrm{g}$, so the outlet is two-phase.

$$
\Delta \mathrm{H}^{\mathrm{vap}}=2256 \mathrm{~J} / \mathrm{g} \text { and find } \mathrm{H}^{\text {satL }}=419.2 \mathrm{~J} / \mathrm{g}
$$

$$
H=q \Delta H^{v a p}+H^{\text {SatL }}
$$

$$
\Rightarrow 2570.64=q^{*}(2256)+419.2
$$

$$
\Rightarrow \mathbf{q}=0.954
$$

(P2.15)as in example 2.6 about transformation of kinetic energy.
$\Rightarrow$ the simplified energy balance is:
$\Delta H=\frac{-\Delta v^{2}}{2 g_{C}}$
$\Rightarrow \Delta H=\frac{-v_{1}^{2}}{2 g_{C}}\left(\left(\frac{D_{1}}{D_{2}}\right)^{4}-1\right)$
$\therefore$ if $: D_{2} / D_{1}=\infty \Rightarrow \Delta H=18 \mathrm{~J} / \mathrm{kg}$

$$
\begin{aligned}
& \left(\frac{C p}{R}\right)_{\text {water }}=4.041 \Rightarrow C p=4184 \mathrm{~J} / \mathrm{kg}-\text { moleK } \\
& \&\left(\frac{C p}{R}\right)_{\text {nitrogen }}=3.5 \Rightarrow C p=3623.9 \mathrm{~J} / \mathrm{kg}-\text { moleK } \\
& \therefore \Delta H=C p(\Delta T) \Rightarrow \Delta T_{\text {nitrogen }}=\frac{\Delta H}{C p}=\frac{18 \mathrm{~J} / \mathrm{kg}-\mathrm{mole}}{3623.9 \mathrm{~J} / \mathrm{kgmoleK}}=0.00496 \mathrm{~K} \\
& \Rightarrow 0.00496 \mathrm{~K}-0.004 \mathrm{~K}=\Delta T_{\text {rise }} \\
& \Rightarrow \Delta T_{\text {rise }}=\max \approx 0.001 \mathrm{~K}
\end{aligned}
$$

(P2.16) Steady state, open system.
$\Rightarrow 0=\dot{m}^{\text {in }} H^{\text {in }}-\dot{m}^{\text {out }} H^{\text {out }}+\underline{Q}$

$$
\begin{aligned}
& \text { a) } \mathrm{P} 1=15 \mathrm{MPa}, \mathrm{~T} 1=600^{\circ} \mathrm{C} \quad \Rightarrow \mathrm{H} 1=3583.1 \mathrm{~kJ} / \mathrm{kg} \quad \text { (Steam Table) } \\
& \mathrm{P} 2=10 \mathrm{MPa}, \mathrm{~T} 2=700^{\circ} \mathrm{C} \quad \Rightarrow \mathrm{H} 2=3870 \mathrm{~kJ} / \mathrm{kg} \quad \text { (Steam Table) } \\
& \Delta \mathrm{H}=\mathrm{Heat}=\mathrm{Q}=\mathrm{H} 2-\mathrm{H} 1=-287 \mathrm{~kJ} / \mathrm{kg}
\end{aligned}
$$

b) for $H^{i g}$ take lowest P at $600^{\circ} \mathrm{C}$ and 0.01 MPa

$$
\mathrm{H}^{\mathrm{ig}}=3706 \mathrm{~J} / \mathrm{g} \quad\left(\text { at } 600^{\circ} \mathrm{C} \text { and } 0.1 \mathrm{MPa}\right)
$$

$$
\frac{H^{i n}-H^{i g}}{R^{*} T^{i n}}=\frac{(3583.1 \mathrm{~J} / \mathrm{g}-3706 \mathrm{~J} / \mathrm{g}) * 18 \mathrm{~g} / \mathrm{mole}}{8.314 \mathrm{~J} / \text { mole }-K^{*}(600+273.15) \mathrm{K}}=\mathbf{- 0 . 3 0 5}
$$

(P2.17) There are several ways to look at this problem, and some choices of system boundries are more difficult than others. One method is presented.
(a). System: piston, donote with subscript p
closed system,
no Q (neglect)
no flow (rigid body)
no change of internal energy (piston doesn't change T)

$m_{p} \frac{g}{g_{C}} \Delta z+m_{p} \frac{\Delta v^{2}}{2 g_{C}}=\underline{W}_{S, p}$
surface forces cause movement; treat this as Ws.

Now we need to determine Ws. The work done by the gas (subscript $g$ ) and the atmosphere (subscript $a$ ) will be equal to the work done on the piston. The gas and atmosphere are closed systems that change size.

System: gas
$\underline{W}_{E C, g}=-\int P_{g} d \underline{V}_{g}$
System: atmosphere.
Note: $d \underline{V}_{g}=-d \underline{V}_{a}$
$\underline{W}_{E C, a}=-\int P_{a} d \underline{V}_{a}=\int P_{a} d \underline{V}_{g}$
Combining work interaction at boundary
$\underline{W}_{s, p}=-\underline{W}_{E C, g}-\underline{W}_{E C, a}$
Which results in the working equation.

$$
\begin{align*}
& m_{p} \frac{g}{g_{C}} \Delta z+m_{p} \frac{\Delta v^{2}}{2 g_{C}}=\underline{W}_{S, p}=\int P_{g} d \underline{V}_{g}-\int P_{a} d \underline{V}_{g} \ldots \ldots . . . . . . . . . . . . . . . . . . .(4) \\
& \underline{W}_{S, p}=n_{g} \int(R T / V) d V-\int P_{a} d \underline{V} \\
& \underline{W}_{S, p}=\left(\mathrm{n}_{\mathrm{g}} \mathrm{RT} 1^{*} \ln \frac{V 2}{V 1}-0.1^{*}(0.25-0.1)\right)=\left(\mathrm{P} 1 \mathrm{~V} 1 * \ln \frac{0.25}{0.1}-\left(0.1^{*} .015\right)\right) . \tag{5}
\end{align*}
$$

$\underline{W}_{S, p}=(1.0 * 0.1 * \ln 2.5-.015) * 1 \mathrm{E}^{2} \mathrm{~cm}^{3} / \mathrm{m}^{3}=76629 \mathrm{~J}$
From E. Bal.
$\mathrm{mv}^{2} / 2 \mathrm{~g}_{\mathrm{c}}=\underline{W}_{S, p}-\mathrm{mg} \Delta \mathrm{z} / \mathrm{g}_{\mathrm{c}}=76629 \mathrm{~J}-700 * 9.81 *(2.5 \mathrm{~m}-1 \mathrm{~m})=66328.6 \mathrm{~J}$
$\mathrm{v}=13.76 \mathrm{~m} / \mathrm{s}$
(b) Free flight, system: Piston, closed system, constant T.

Equation (1) applies where $\underline{W}_{s, p}=0$ (no surface forces acting on piston, because we will ignore air resistance). The kinetic energy will go to zero at the top of flight. The initial kinetic energy from part (a).
$m_{p} \frac{g}{g_{C}} \Delta z+m_{p} \frac{\Delta v^{2}}{2 g_{C}}=700(9.81) \Delta z-66328.6 \mathrm{~J}=0$.
$\Delta z=9.6 m$
(not counting 2.5 meters of tube).
(c) $\quad \mathrm{P}_{\mathrm{g}} \mathrm{V}_{\mathrm{g}}=$ constant, since isothermal, $\mathrm{P} 2=\mathrm{P} 1 \mathrm{~V} 1 / \mathrm{V} 2=\mathbf{4}$ bar
(d) $\quad \frac{\underline{V}_{2}}{\underline{V}_{1}}=5.88, \Rightarrow \underline{V}_{2}=0.588$ plugging into (5)

$$
\underline{W}_{\mathrm{s}, \mathrm{p}}=(1.0 * 0.1 * \ln 5.88-0.1 *(0.588-0.1)) * 1 \mathrm{E} 6 \mathrm{~cm}^{3} / \mathrm{m}^{3}
$$

$\underline{W}_{\mathrm{s}, \mathrm{P}}=128350 \mathrm{~J}$
Using (6)
$\mathrm{mv}^{2} / 2 \mathrm{~g}_{\mathrm{c}}=128350-700 * 9.81(5.88-1)=\mathbf{9 4 8 4 0} \mathbf{~ J}$
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plugging into (7)

$$
\begin{aligned}
& \Rightarrow 700 * 9.81 * \Delta z=94840 \mathrm{~J}, \\
& \Rightarrow \Delta z=13.8 \mathrm{~m}, \text { free, flight }
\end{aligned}
$$

(e) the piston will accelerate when the upword force (Fup) > down ward force (Fdown). The piston will decelerate when Fup< Fdown. The maximum exit velocity will be obtained if the piston leaves the cylinder at the condition Fup = Fdown.

$$
\begin{align*}
& F_{u p}=P_{2} A \\
& F_{\text {down }}=P_{a} A+\frac{m g}{g_{C}} \\
& P_{2} A=P_{a} A+\frac{m g}{g_{C}} \ldots \tag{8}
\end{align*}
$$

$\mathrm{P}_{2}$ can be related to $\mathrm{V}_{2}$ since gas is isothermal, $\mathrm{PV}_{\mathrm{g}}=$ constant,
$P_{2}=\frac{P_{1} V_{1}}{V_{2}}$.
Substituting (9) into (8), and inserting the values.
$(1 E 6 P a)\left(V_{1} / V_{2}\right)\left(0.1 m^{2}\right)=(0.1 E 6 P a)\left(0.1 m^{2}\right)+700 * 9.81$
$\frac{\underline{V}_{2}}{\underline{V}_{1}}=5.93, \Rightarrow \underline{V}_{2}=0.593 \mathrm{~m}^{3}$
Answer will be almost the same as (d)
Plugging into (5)
$\underline{W}_{\mathrm{S}, \mathrm{p}}=\left(1.0 * 0.1^{*} \ln 5.93-0.1 *(0.593-0.1)\right) * 1 \mathrm{E} 6 \mathrm{~cm}^{3} / \mathrm{m}^{3}$
$\underline{W}_{\mathrm{s}, \mathrm{p}}=128700 \mathrm{~J}$
Using (6),
$\mathrm{mu}^{2} / 2 \mathrm{~g}_{\mathrm{c}}=128700-700 * 9.81(5.93-1)=\mathbf{9 4 8 4 6} \mathbf{~ J}$
sub. Into (7)
$\Rightarrow 700 * 9.81 * \Delta z=94846 \mathrm{~J}$,
$\Rightarrow \Delta z=13.8 \mathrm{~m}$, free, flight
(P2.18)


Initial and final conditions , $\mathrm{P} 1=50 \mathrm{psig}=64.7 \mathrm{psia}=\mathbf{0 . 4 4 6 2} \mathrm{MPa}$.
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© J.R. Elliott, C.T. Lira, 2001-2014, all rights reserved. (12/4/2014)
ball: $3 \mathrm{in} * 2.54 \mathrm{~cm} / \mathrm{in}=7.62 \mathrm{~cm}$ dia
cylinder volume: V2=100 $\mathrm{cm} * 3.14 *(7.62 / 2)^{2}=4558.1 \mathrm{~cm}^{3}$
ball mass: $0.125 \mathrm{lbm} * .454 \mathrm{~kg} / \mathrm{lbm}=.057 \mathrm{~kg}$;
exit velocity: $40 \mathrm{mi} / \mathrm{hr} * 5280 \mathrm{ft} / \mathrm{mi}^{*} 0.3048 \mathrm{~m} / \mathrm{ft} * 1 \mathrm{hr} / 3600 \mathrm{sec}=\mathbf{1 7 . 8 8} \mathbf{~ m} / \mathbf{s}$
Energy Balance:
see E. Bal. From (P2.17), except potential energy is not included since piston is horizontal, ball replaces the piston of (P2.17).
adapting (4), (5) from (P.2.17)
$\mathrm{W}_{\mathrm{S}, \text { ball }}=0.057 / 2 *(17.88)^{2}=9.11 \mathbf{~ J}$
$\mathrm{m} \Delta \mathrm{v}^{2} / 2 \mathrm{~g}_{\mathrm{c}}=\mathrm{W}_{\mathrm{S}, \text { ball }}=0.4462 * \mathrm{~V} 1 * \ln (\mathrm{~V} 2 / \mathrm{V} 1)-0.1 *(4558.1-\mathrm{V} 1)=\mathbf{9 . 1 1} \mathbf{J}$
Using solver $\Rightarrow \mathrm{V} 1=387 \mathrm{~cm}^{3}$ not counting Vball.
(P2.19)


Description: Forces are imbalanced. The piston will accelerate to the right until forces are balanced, then decelerate to right (accel to left) until velocity is zero. Forces will then be imbalanced and the piston will move back to left until original location is reached. In absence of friction or viscous dissipation, motion will be perpetual. We are asked to assume isothermal. Actual process is probably closer to adiabatic because it is so fast heat transfer is unlikely. Nevertheless, let us assume isothermal. Heat must be added to the left side and removed from the right side to keep isothermal.
$\mathrm{P} \underline{\mathrm{V}}=\mathrm{nRT}$ for each side and will be constant for each side if isothermal because each side is closed.

Overall E-bal around both sides:

$$
\begin{equation*}
\Delta \underline{U}^{L}+\Delta \underline{U}^{R}+\Delta K E_{p i s t o n}=\underline{Q}^{L}+\underline{Q}^{R} \tag{1}
\end{equation*}
$$

(a) $\Delta U^{L}=0$ and $\Delta U^{R}=0$. When piston stops the first time, $\Delta K E_{\text {piston }}=0$, because $v=0$ at the beginning and end of motion. Therefore by the energy balance, $\underline{Q}^{L}+\underline{Q}^{R}=0$ from (1).

$$
\begin{equation*}
\underline{Q}=n R T \ln \left(\underline{V}^{f} / \underline{V}^{i}\right) \text { for each side } \tag{2}
\end{equation*}
$$

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$$
\begin{equation*}
n^{L} R T^{L} \ln \frac{\underline{V}^{i, L}+\Delta \underline{V}^{L}}{\underline{V}^{i, L}}+n^{R} R T^{R} \ln \frac{\underline{V}^{i, R}-\Delta \underline{V}^{L}}{\underline{V}^{i, R}}=0 \tag{3}
\end{equation*}
$$

$$
\begin{aligned}
& n^{L} R T^{L}=P^{L} V^{L}=0.8 \mathrm{MPa}\left(10^{5} \mathrm{~cm}^{3}\right)=8 \mathrm{E} 4 \mathrm{~J} \\
& n^{R} R T^{R}=P^{R} V^{R}=0.1 \mathrm{MPa}\left(1.5 \mathrm{E} 5 \mathrm{~cm}^{3}\right)=1.5 \mathrm{E} 4 \mathrm{~J}
\end{aligned}
$$

Eqn (3) becomes

$$
8 \ln \frac{0.1+\Delta \underline{V}^{L}}{0.1}+1.5 \ln \frac{0.15-\Delta \underline{V}^{L}}{0.15}=0
$$

Solving by trial and error $\Delta \underline{V}^{L}=0.1488 \mathrm{~m}^{3}$.

$$
\begin{aligned}
& \underline{V}^{L}=0.1+0.1488=0.2488 \mathrm{~m}^{3}=248,800 \mathrm{~cm}^{3} . \\
& \underline{V}^{R}=0.15-0.1488=0.0012 \mathrm{~m}^{3}=1200 \mathrm{~cm}^{3} . \\
& \underline{P}^{R}=1.5 \mathrm{E} 4 \mathrm{~J} / 1200 \mathrm{~cm}^{3}=12.5 \mathrm{MPa} ; P^{L}=8 \mathrm{E} 4 \mathrm{~J} / 248,800 \mathrm{~cm}^{3}=0.32 \mathrm{MPa}
\end{aligned}
$$

(b) $\Delta U^{L}=0$ and $\Delta U^{R}=0$ still applies. Eqn (1) becomes

$$
\begin{equation*}
\Delta K E_{\text {piston }}=\underline{Q}^{L}+\underline{Q}^{R} \tag{4}
\end{equation*}
$$

Eqn (2) still applies. Force balance:

$$
P^{L}=\frac{n^{L} R T^{L}}{\underline{V}^{i, L}+\Delta \underline{V}^{L}}=P^{R}=\frac{n^{R} R T^{R}}{\underline{V}^{i, R}-\Delta \underline{V}^{L}}
$$

cross multiply

$$
\begin{gathered}
8 E 4\left(0.15-\Delta \underline{V}^{L}\right)=1.5 E 4\left(0.1+\Delta \underline{V}^{L}\right) \mathrm{Jm}^{3} \\
8(0.15)-1.5(0.1)=(1.5+8) \Delta \underline{V}^{L} \\
1.05=9.5 \Delta \underline{V}^{L} \\
\Delta \underline{V}^{L}=0.1105 \mathrm{~m}^{3}
\end{gathered}
$$

Eqn (1) becomes

$$
\begin{equation*}
\Delta K E_{\text {piston }}=8 \ln \frac{0.1+\Delta \underline{V}^{L}}{0.1}+1.5 \ln \frac{0.15-\Delta \underline{V}^{L}}{0.15} \tag{5}
\end{equation*}
$$

(Note: another way to find the same $\Delta \underline{V}^{L}$ is to maximize $\Delta K E$ by trial and error or by differentiating to look for the maximum.)

Inserting $\Delta \underline{V}^{L}=0.1105$
$\Delta K E_{\text {piston }}=1.717 E 4 \mathrm{~J}=1 / 2 m v^{2} \quad\left(\right.$ note $\left.v^{i}=0\right)$

$$
v=\sqrt{\frac{2(1.717 E 4) J}{700 \mathrm{~kg}}}=7 \mathrm{~m} / \mathrm{s}
$$

We can show that the pressures are balanced:

$$
\begin{aligned}
& \underline{V}^{L}=0.1+0.1105=0.2105 \mathrm{~m}^{3}=210,500 \mathrm{~cm}^{3} . \\
& \underline{V}^{R}=0.15-0.1105=0.0395 \mathrm{~m}^{3}=39,500 \mathrm{~cm}^{3} . \\
& P^{R}=1.5 \mathrm{E} 4 \mathrm{~J} / 39,500 \mathrm{~cm}^{3}=0.38 \mathrm{MPa} ; P^{L}=8 \mathrm{E} 4 \mathrm{~J} / 210,500 \mathrm{~cm}^{3}=0.38 \mathrm{MPa}
\end{aligned}
$$

