(P4.1)

(a) the number of microstates is 2^N

(b) 3 particles total

$\Rightarrow p_{\{2H,1T\}} = \dfrac{3!}{2!*1!} = 3$ number microstates of specific arrangement (macrostate)

probability = (# microstates of specific arrangement)/(total # of microstates)

$$prob = \frac{3}{2^3} = \frac{3}{8}$$

(c) # microstates.

$$p_{\{2H,2T\}} = \frac{4!}{2!*2!} = 6$$

$$p_{\{3H,2T\}} = \frac{5!}{3!*2!} = 10$$

$$p_{\{4H,2T\}} = \frac{6!}{4!*2!} = 15$$

$$p_{\{3H,3T\}} = \frac{6!}{3!*3!} = 20$$

(d)

macrostate		# of microstates*
H	T	
0	8	1
1	7	8
2	6	28
3	5	56
4	4	70
5	3	56
6	2	28
7	1	8
8	0	1

* number of microstates = $\dfrac{8!}{m!(8-m)!}$

total number of microstates is 2^8 = 256, which is the same as the sum from the table.

portion of microstates (probability) for requested configurations:

{5:3} = 56/256 = 0.219 = 22%

{4:4} = 70/256 = 0.273 = 27%

{3:5) = 22% like {5:3}

probability of any one of the three most evenly distributed states = 22% + 27% + 22% = 71%

(e) for 8 particle system, Stirling's approx will not apply

$\Delta \underline{S}/k = \ln(p\{4:4\}/p\{5:3\}) = \ln(70/56) = 0.223$

(P4.2) Initial (each x represents 5 molecule)

xxxx	

Final

x	x
x	x

Create a space with a three empty boxes for the initial state. The number of molecules is too small to use Stirling's approximation.

p1 = 20!/(20!0!0!0!) = 1

p2 = 20!/(5!5!5!5!) = 20*19*18*17*16*15*14*13*12*11*10*9*8*7*6/(5*4*3*2)^3 = 11732745024

ΔS/k = ln(p2/p1) = ln(11732745024) = 23.18

(P4.3) 15 molecules in 3 boxes, molecules are identical

$$p_j = \frac{N!}{\prod_{i=1} m_{ij}!} \quad \text{...Eqn. 4.4}$$

$$p_1 = \frac{15!}{9!4!2!} = 75075$$

$$p_2 = \frac{15!}{(5!)^3} = 756756$$

$$\frac{\Delta \underline{S}}{k} = \ln\left[\frac{p_2}{p_1}\right] = 2.31$$

(P4.4) two dice.

$\frac{\Delta S}{k} = ??$ for going from double sixes to a four and three.

$\Rightarrow$ for double sixes, we have probability of 1/6 for each dice.

$$\Rightarrow p_1 = \frac{2!}{\left(\frac{1}{6}\right)*\left(\frac{1}{6}\right)}$$

for one four and one three $\Rightarrow$ probability applied for 1/6 for each one in each dice,

$$\Rightarrow p_2 = \frac{2!}{\left(\frac{1}{6}\right)*\left(\frac{1}{6}\right)} * 2$$

$$\frac{\Delta S}{k} = \ln\left(\frac{p_2}{p_1}\right) = \ln 2 = 0.693$$

(P4.5) ΔS = ??

Assume Nitrogen is an Ideal gas $\Rightarrow$ $PV = RT$ .... Eqn. 1.12

$$\Rightarrow P_1 = \frac{8.314\left(cm^3 * MPa / mole - K\right) * 300K}{23\left(L/mole\right) * (1000cm^3/1L)} = 0.108MPa$$

Similarly $\Rightarrow P_2 = 0.00723MPa$

$$\Delta S = Cp \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} \quad \text{.. Eqn. 4.29}$$

$$Cp = \frac{7R}{2} \text{ .....(ig)}$$

$$\Delta S = \frac{7R}{2} * \ln\frac{400}{300} - 8.314 * \ln\frac{0.00723}{0.108} = 30.88J/mole - K = 1.07kJ/kg - K$$

(P4.6) (a) m-balance: $dn^{in} = -dn^{out}$

S-balance:

$$\frac{d(nS)^{in}}{dt} = -S^{out}\frac{dn^{out}}{dt} \Rightarrow n^{in}dS^{in} + S^{in}dn^{in} = -S^{out}dn^{out}$$

But physically, we know that the leaking fluid is at the same state as the fluid in the tank; therefore, the S-balance becomes:

$(ndS + Sdn)^{inside} = -(Sdn)^{out}$, and $dn^{inside} = -dn^{out}$ so $\Delta S = 0$

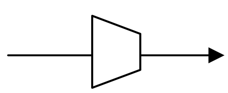
from the steam table .

State	P(Mpa)	$T^{O}C$	H(kJ/kg)	S(kJ/kg*K)
1(in)	1	400	3264.5	7.4669
2 (out)	0.1	***120.8***	2717.86	7.4669

At 1 bar =	0.1 MPa
T	S
100	7.361
120.8	**7.4669**
150	7.6148

By interpolation, implies **T = 120.8^{O}C**

(P4.7) (a) Steady-state flow, ΔH = Ws



Start 1 mole basis:

$x_1 = 0.333, x_2 = 0.667, adiabatic, Cp = x_1Cp_1 + x_2Cp_2$, Cp for each is the same anyway.

$MW = x_1MW_1 + x_2MW_2 = 0.333(12+16) + 0.667 * 2 = 10.66(g/mole)$

R = 1.987BTU/lbmol-R.

$$\Delta H = W_S = \int_{T_1}^{T_2} CpdT = \frac{7}{2} * R * \left(1100 - 100\right)^{\circ}R$$

$$\Rightarrow \Delta H = 6954.5BTU/lbmol$$

$$\& \dot{m} = 1ton/h = 2000lb/h.$$

$$\& MW = 10.66lb/lbmol$$

$$\Rightarrow \Delta H = \frac{2000lb}{h} * \frac{lbmol}{10.66lb} * \frac{6954.5BTU}{lbmol}$$

$$\Rightarrow \Delta H = W_S = 1{,}305{,}000 = 1.3 * 10^6 BTU/h$$

(b) $\eta = ??$ of the compressor.

To find the efficiency of the compressor, $\Rightarrow S_1 = S_2$

But the enthalpy and the internal energy will change which gives a change in the

Work. $\Rightarrow \eta = \dfrac{W_S{}'}{W_S} = ??$

$$\Delta S = 0 = Cp\ln\frac{T_2'}{T_1} - R\ln\frac{P_2}{P_1}$$

$$\Rightarrow Cp\ln\frac{T_2'}{T_1} = R\ln\frac{P_2}{P_1}$$

$$\Rightarrow \left(\frac{T_2'}{T_1}\right)^{Cp} = \left(\frac{P_2}{P_1}\right)^{R}$$

$$\Rightarrow T_2' = \left(\frac{P_2}{P_1}\right)^{\frac{R}{Cp}} * T_1$$

$$\Rightarrow T_2' = \left(\frac{100}{5}\right)^{\frac{2}{7}} * 559R$$

$$T_2' = 1315R$$

$$\& \Delta H' = Cp(T_2' - T_1) = 6.95(1315 - 559) \quad \Rightarrow \eta = \frac{\Delta H'}{\Delta H} = \frac{5258}{6955} = 0.76$$

$$\Rightarrow \Delta H' = 5258BTU/lbmol$$

(P4.8) Adiabatic, steady-state open system $\Rightarrow$ Q = 0, &(Cp/R = 7/2)………………………… ig

$$W = \int_{300}^{625} CpdT = \frac{7R}{2} * (625 - 300) = 9457.175kJ/kmole * \frac{1kmole}{28kg} = 337.76kJ/kg$$

$$\eta = ??$$

$$\Delta S = 0 \Rightarrow \frac{T_2'}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{R}{Cp}}$$

$$\Rightarrow T_2' = 533.5K$$

$$\Rightarrow \Delta H' = Cp\left(T_2' - T_1\right) = \left(\frac{7*8.314}{2}\right) * \left(533.5 - 300\right)$$

$$\Rightarrow \Delta H' = 6794.77kJ/mol$$

$$\Rightarrow \eta = \frac{\Delta H'}{\Delta H} = \frac{6794.77kJ/kmol}{337.76kJ/kg * 28kg/kmol} = 0.718$$

$$\Rightarrow \eta = 71.8\%$$

(P4.9) work required per kg of steam through this compressor?

By looking at the steam table in the back of the book

P(MPa)	T(^{O}C)	H(kJ/kg)	S(kJ/kg-K)

0.8	200	2839.7	6.8176
4	500	3446	7.0922

$W = \Delta H = 3446 - 2839.7 = 606.3 kJ/kg$

now find W' = ??

ΔS = 0 (reversible), ⇒ look in the steam table (@P = 4.0MPa) to find a similar value for S = 6.8176kJ/kg-K, if this value is not available so find it by interpolation.

H(kJ/kg)	S(kJ/kg-K)
3214.5	6.7714
H' = ??	S' = 6.8176
3331.2	6.9386

$$\Rightarrow \frac{3331.5 - H'}{3331.2 - 3214.5} = \frac{6.9386 - 6.8176}{6.9386 - 6.7714}, \Rightarrow H' = 3246.7$$

$$\Rightarrow \Delta H' = W' = 3246.7 - 2839.7 = 407 kJ/kg$$

$$\Rightarrow \eta = \frac{407}{606.3} = 0.67, \Rightarrow \eta = 67\%$$

(P4.10)@ P = 2.0 MPa & T = 600^{O}C, ⇒ H = 3690.7 kJ/kg, S = 7.7043kJ/kg-K (Steam table)

	T^{O}C	H_L(kJ/kg)	ΔH^{vap}(kJ/kg)
steam table	20	83.91	2453.52
Interpolation	**24**	**100.646**	**2444.098**
steam table	24	104.83	2441.68

$$H = H_L + q(\Delta H^{Vap}) = 1006.46 + 0.98 * (2441.68) = 2493k.49J/kg$$

$$\Rightarrow W_S = \Delta H = 3690.7 - 2493.49 = 1197.21 kJ/kg$$

$$\eta = ?? \ , \quad \eta = \frac{\Delta H}{\Delta H'} = \frac{W}{W'},$$

⇒ ΔS = 0 (reversible), ⇒look for S in the sat'd temp. steam table and find H by interpolation, ⇒ $W' = 1408.0 kJ/kg$

$$\Rightarrow \eta = \frac{1197.2}{1408.0} = 0.8503, \Rightarrow \eta = 85\%$$

(P4.11)

$$P_1 = 0.1 MPa, Sat'd_{vap}$$

$$P_2 = 10 MPa$$

$$T_2 = 1100^O C$$

State	**P(MPa)**	**T(^{O}C)**	**H(kJ/kg)**	**S(kJ/kg-K)**

1	0.1	99.61	2674.95	7.3589
2'	10		**4062.53**	*7.3589*
2	10	1100	4870.3	8.0288

interpolation for above table:

H'_2 = 4062.53	(interpolation)
H(kJ/kg)	**S(kJ/kg-K)**
3992	7.2916
4062.53	*7.3589*
4114.5	7.4085

$\Rightarrow \Delta H = W_S = 4870.3 - 2674.95 = 2195.35 kJ/kg$

mass flow rate = 1 kg/s

$\Rightarrow \dot{W}_S = 2195.35 kJ/s = 2195350 watt$

$\& \, 1 watt = 0.001341022 hp$

$\Rightarrow \dot{W}_S = 2944.01 hp$

$\& \, \Delta H' = 4062.53 - 2674.95 = 1387.58 kJ/kg$

$$\Rightarrow \eta = \frac{\Delta H'}{\Delta H} = \frac{1387.58}{2195.35} = 0.63$$

$\Rightarrow \eta = 63.2\%$

(P4.12)

Ebal: $\Delta H = W$.

Sbal: $\Delta S^{rev} = 0$ => $\left(\frac{T_2^{rev}}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\left(\frac{R}{Cp}\right)}$ => T_2^{rev} = (20+273)*18^(8.314/44) = 506K.

$W^{rev} = C_p(T_2^{rev} - T_1)$ = 44*(506-293) = 9372J/mol => W_{act} = 9372/0.85 = 13.4kJ/mol

$W^{act} = C_p(T_2^{act} - T_1)$ = 13400 => T_2^{act} = (13400/44)+293 = 597K

(P4.15) Through the valve $\Rightarrow H^{in} = H^{out}$

$P^{in} = 3MPa \quad P^{out} = 0.1MPa \qquad T_{out} = 110^{O}C = 383.15K$

(By interpolation) Find H^{out} from steam table.

$$\frac{150-110}{150-100} = \frac{2776.6 - H^{out}}{2776.6 - 2675.8}$$

$\Rightarrow$ H^{out} **= 2695.96 kJ/kg**

At 3MPa table use same value for $\boldsymbol{H^{in}}$ to find $\boldsymbol{S^{in}}$

$\Rightarrow$ By interpolation $\dfrac{2856.5 - 2695.96}{2856.5 - 2803.2} = \dfrac{6.2893 - S^{in}}{6.2893 - 6.1856}$

$\Rightarrow$ $\boldsymbol{S^{in}}$ **= 5.976kJ/kg-K**

The process should be irreversible. To find S^{out}, interpolate using temperature at 0.1 MPa:

$$\frac{150-110}{150-100}=\frac{7.6148-S^{out}}{7.6148-7.3610}$$

S^{out} = 7.4118 kJ/kg-K, since S^{out}>S^{in} entropy has been generated. The entropy balance is:

$$0 = S^{in}\dot{m}^{in} - S^{out}\dot{m}^{out} + \underline{\dot{S}}_{gen}$$

(P4.18) An insulated cylinder is fitted with a freely floating piston, ...

Steam 0.5kg, P^i= 9 bars, q=0.9, goes to satd vapor at 30 bars, $\underline{W}_{air}$ = -360 kJ.
The volume change of the air is equal and opposite the volume change of the steam. The volume change of the steam is (using sat properties at 0.9 MPa)
$\underline{V}^i_{steam}$ = 0.5kg (V^{satL} + q (V^{satV} – V^{satL}))m^3/kg = 0.5(0.001121 + 0.9(0.2149-.001121)) = 0.09676m^3
$\underline{V}^f_{steam}$ = 0.5*V^{satV}= 0.5kg*.0667m^3/kg = 0.03335 m^3;
$\Delta\underline{V}_{steam}$ = 0.03335 – 0.09876 = - 0.06541 m^3.
$\underline{V}^f_{air}$ = $\underline{V}^i_{air}$ + $\Delta\underline{V}_{air}$ = 0.05 + 0.06541 = 0.1154 m^3
Because we are not told the mass and area of the piston, let us consider it massless, so that the initial pressure of the air is 9 bar and the final pressure is 30bar. The air will be treated like an ideal gas.
$d(nU)=H^{in}dn + dWair$; integrate term-by-term
$n^fU^f - n^iU^i = H^{in}(n^f - n^i)$ -360
for the air, let H_R = 0 for 50 bar and 300K, the inlet condition.
$n^i = P\underline{V}/RT$ = 0.9MPa (0.05m^3)(10^{6}cm^3/m^3)/8.31447 (cm^3-MPa)/300 = 18.041 moles
$n^f = P\underline{V}/RT$ = 3MPa(0.1154m^3)(10^{6}cm^3/m^3)/8.31447 (cm^3-MPa)/T^f_{air} = 41638/T^f_{air} moles
Let H_R =0 at 50 bar and 300K ⇒ $U_R = H_R\text{-}(PV)=0\text{-}RT_R$ = -2494.3 J/mol
Because U is independent of P, then U^i= U_R = -2494.3 J/mol.
For air, use Cv = 2.5R = 20.786 J/mol-K
The energy balance becomes
41638/T^f_{air} [20.786(T^f_{air} -300) -2494.3] J= -360000 J
Trial&Error ⇒ T_f = 297 K; the temperature change is very small for this case.