

(P6.1)

(a)

$$\left(\frac{\partial H}{\partial S}\right)_V$$

Expansion Rule: $dH = TdS + VdP$

$$\begin{aligned}\left(\frac{\partial H}{\partial S}\right)_V &= T \left(\frac{\partial S}{\partial S}\right)_V + V \left(\frac{\partial P}{\partial S}\right)_V \\ &= T + V \left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial T}{\partial S}\right)_V \quad (\text{chain rule}) \\ &= T \left(1 + \frac{V}{C_V} \left(\frac{\partial P}{\partial T}\right)_V\right)\end{aligned}$$

(b)

$$\left(\frac{\partial H}{\partial P}\right)_V$$

Expansion rule: $dH = TdS + VdP$

$$\begin{aligned}\left(\frac{\partial H}{\partial P}\right)_V &= T \left(\frac{\partial S}{\partial P}\right)_V + V \left(\frac{\partial P}{\partial P}\right)_V \\ &= T \left(\frac{\partial S}{\partial T}\right)_V \left(\frac{\partial T}{\partial P}\right)_V + V = C_V \left(\frac{\partial T}{\partial P}\right)_V + V\end{aligned}$$

(c)

$$\left(\frac{\partial G}{\partial H}\right)_P$$

chain using T since it is measurable

$$\left(\frac{\partial G}{\partial T}\right)_P \left(\frac{\partial T}{\partial H}\right)_P = \left(\frac{\partial G}{\partial T}\right)_P / \left(\frac{\partial H}{\partial T}\right)_P = \frac{\left(\frac{\partial G}{\partial T}\right)_P}{C_P}$$

Use expansion rule: $dG = -SdT + VdP$

$$\left(\frac{\partial G}{\partial T}\right)_P = -S \left(\frac{\partial T}{\partial T}\right)_P + V \left(\frac{\partial P}{\partial T}\right)_P = -S$$

thus:

$$\left(\frac{\partial G}{\partial H}\right)_P = -\frac{S}{C_P}$$